Horizontal product differentiation

How far does a market extend?
Which firms compete with each other?
What is an industry?

Products are *not* homogeneous.
Exceptions: petrol, electricity.

But some products are more equal to each other than to other products in the economy. These products constitute an industry.

A market with *product differentiation*.

But: where do we draw the line?

Example:

- beer vs. soda?
- soda vs. milk?

- beer vs. milk?

Two kinds of product differentiation

(i) Horizontal differentiation: Consumers differ in their preferences over the product's characteristics. Examples: colour, taste, location of outlet.

(ii) Vertical differentiation: Products differ in some characteristic in which all consumers agree what is best. Call this characteristic quality. (*quality competition*)

Horizontal differentiation

Two questions:

1. Is the product variation too large in equilibrium?

2. Are there too many variants in equilibrium?

Question 1: A fixed number of firms. Which product variants will they choose?

Question 2: Variation is maximal. How many firms will enter the market?

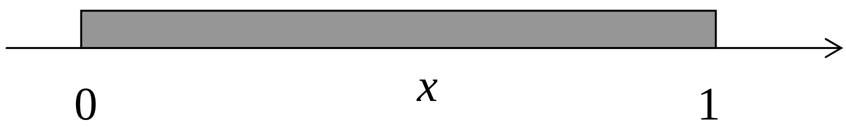
The two questions call for different models.

Variation in equilibrium

Will products supplied in an unregulated market be too similar or too different, relative to social optimum?

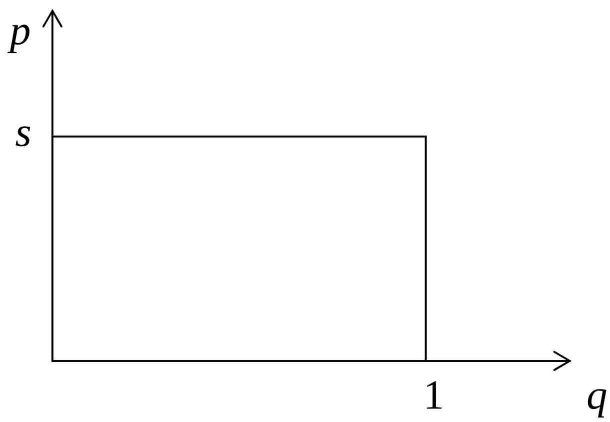
Hotelling (1929)

Product space: the line segment [0, 1].
Two firms: one at 0, one at 1.



Consumers are uniformly distributed along [0, 1].
A consumer at x prefers the product variety x.

Consumers have unit demand:



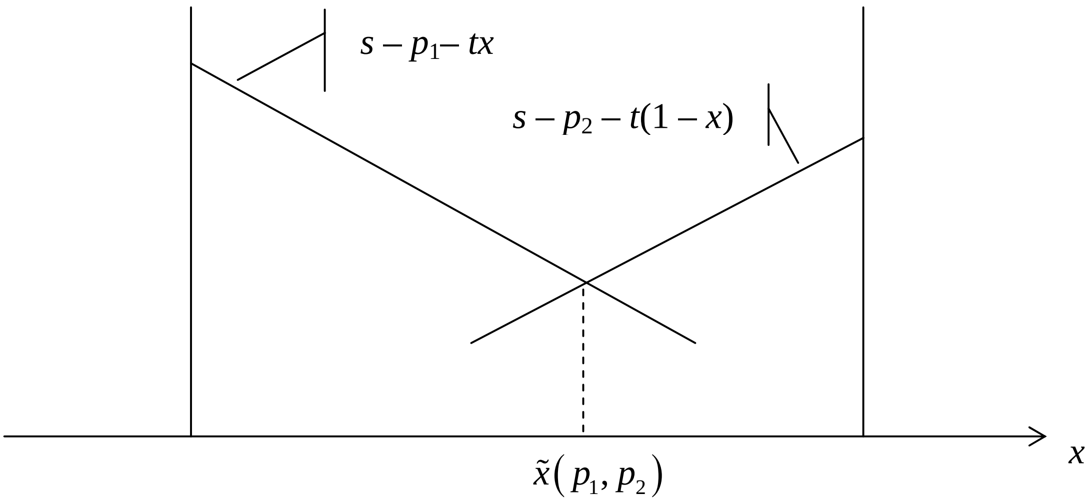
Disutility from consuming product variety *y*:

$t(|y - x|)$ – ‘‘transportation costs”

Linear transportation costs: $t(d) = td$

Generalised prices (with firm 1 at 0 and firm 2 at 1):

$p_1 + tx$ and $p_2 + t(1 - x)$



The indifferent consumer: $\tilde{x}$

$s - p_1 - t\tilde{x} = s - p_2 - t(1 - \tilde{x})$.

$$\Rightarrow \tilde{x}(p_1, p_2) = \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

[But check that: (i) $0 \le \tilde{x} \le 1$; (ii) $\tilde{x}$ wants to buy.]

Normalizing the number of consumers: N = 1 (thousand)

$$D_1(p_1, p_2) = \tilde{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

$$D_2(p_1, p_2) = 1 - \tilde{x} = \frac{1}{2} + \frac{p_1 - p_2}{2t}$$

Constant unit cost of production: c

$$\pi_1(p_1, p_2) = (p_1 - c)\left[\frac{1}{2} + \frac{p_2 - p_1}{2t}\right]$$

Price competition.

Equilibrium conditions: $\frac{\partial \pi_1}{\partial p_1} = 0; \ \frac{\partial \pi_2}{\partial p_2} = 0$

FOC[1]:

$$\underbrace{(p_1 - c)\left(-\frac{1}{2t}\right)}_{\text{increased price reduces sales}} + \underbrace{\frac{1}{2} + \frac{p_2 - p_1}{2t}}_{\text{increased price increases gain per unit sold}} = 0$$

$\Rightarrow$ FOC[1]: $2p_1 - p_2 = c + t$

FOC[2]: $2p_2 - p_1 = c + t$

$\Rightarrow$ $p_1^* = p_2^* = c + t$

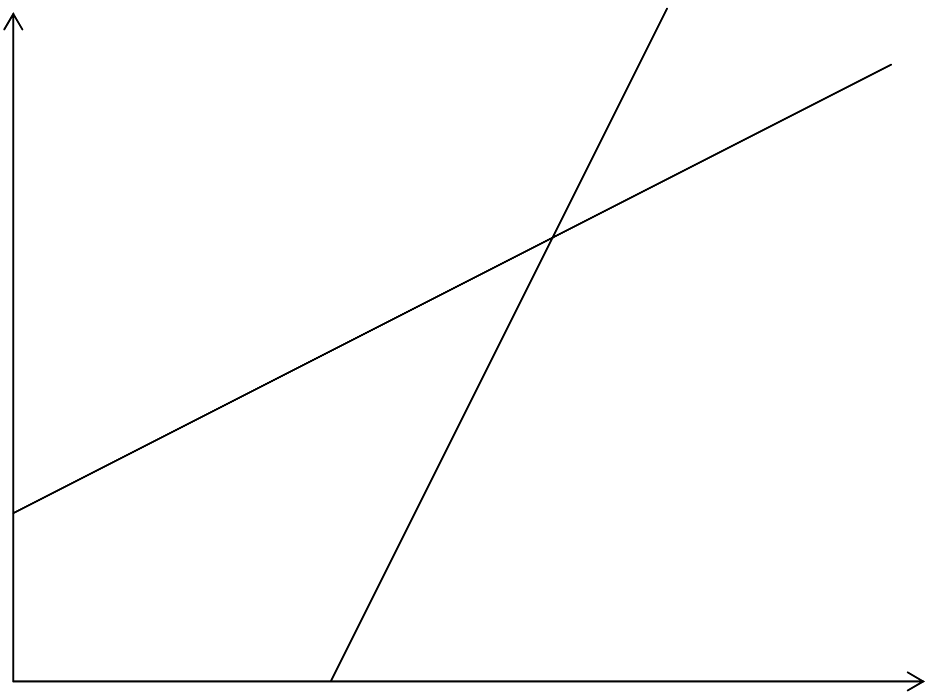
- The indifferent consumer does want to buy if:

$$s \geq c + \tfrac{3}{2}t$$

- Prices are *strategic complements*:

$$\frac{\partial^2 \pi_1}{\partial p_1 \partial p_2} = \frac{1}{2t} > 0$$

Best-response function: $p_1 = \frac{1}{2}(p_2 + c + t)$



The degree of product differentiation: t

Product differentiation makes firms less aggressive in their pricing.

But are 0 and 1 the firms' equilibrium product variations?

Two-stage game of product differentiation:

Stage 1: Firms choose locations on [0, 1].

Stage 2: Firms choose prices.

Linear vs. convex transportation costs.

- Convex costs analytically tractable but economically less meaningful?

Assume quadratic transportation costs.

Stage 2:
Firms 1 and 2 located in a and $1 - b$, $a \geq 0$, $b \geq 0$, $a + b \leq 1$.

The indifferent consumer:

$$p_1 + t(\tilde{x} - a)^2 = p_2 + t(1 - b - \tilde{x})^2$$

$$\tilde{x} = a + \frac{1}{2}\left(1 - a - b\right) + \frac{p_2 - p_1}{2t\left(1 - a - b\right)}$$

$D_1(p_1, p_2) = \tilde{x}$, $D_2(p_1, p_2) = 1 - \tilde{x}$

$$\pi_1\left(p_1, p_2\right) = \left(p_1 - c\right)\left[a + \frac{1}{2}\left(1 - a - b\right) + \frac{p_2 - p_1}{2t\left(1 - a - b\right)}\right]$$

Equilibrium conditions: $\frac{\partial \pi_1}{\partial p_1} = 0$; $\frac{\partial \pi_2}{\partial p_2} = 0$

FOC[1]: $2p_1 - p_2 = c + t(1 - a - b)(1 + a - b)$

FOC[2]: $2p_2 - p_1 = c + t(1 - a - b)(1 - a + b)$

Equilibrium:

$$p_1 = c + t(1 - a - b)\left(1 + \frac{a - b}{3}\right)$$
$$p_2 = c + t(1 - a - b)\left(1 + \frac{b - a}{3}\right)$$

- Symmetric location: $a = b \Rightarrow p_1 = p_2 = c + t(1 - 2a)$
- A firm's price decreases when the other firm gets closer: $\frac{dp_1}{db} < 0$.
- Stage-2 outcome depends on locations:
  $p_1 = p_1(a, b)$, $p_2 = p_2(a, b)$

Stage 1:

$\pi_1(a, b) = [p_1(a, b) - \mathrm{c}]D_1(a, b, p_1(a, b), p_2(a, b))$

$$\frac{d\pi_1}{da} = D_1\frac{\partial p_1}{\partial a} + (p_1 - c)\left[\frac{\partial D_1}{\partial a} + \frac{\partial D_1}{\partial p_1}\frac{\partial p_1}{\partial a} + \frac{\partial D_1}{\partial p_2}\frac{\partial p_2}{\partial a}\right]$$

$$= \underbrace{\left[D_1 + (p_1 - c)\frac{\partial D_1}{\partial p_1}\right]}_{=0}\frac{\partial p_1}{\partial a} + (p_1 - c)\left[\frac{\partial D_1}{\partial a} + \frac{\partial D_1}{\partial p_2}\frac{\partial p_2}{\partial a}\right]$$

$$\frac{d\pi_1}{da} = (p_1 - c)(\underbrace{\frac{\partial D_1}{\partial a}}_{\substack{\text{direct}\\\text{effect;}\\>0}} + \underbrace{\overbrace{\frac{\partial D_1}{\partial p_2}}^{>0}\overbrace{\frac{\partial p_2}{\partial a}}^{<0}}_{\substack{\text{strategic}\\\text{effect;}\\<0}})$$

Moving toward the middle:
A positive direct effect vs. a negative strategic effect.

$$\frac{\partial D_1}{\partial a} = \frac{1}{2} + \frac{p_2 - p_1}{2t(1-a-b)^2} = \frac{1}{2} + \frac{b-a}{3(1-a-b)}$$

$$= \frac{3-5a-b}{6(1-a-b)} > 0, \text{if } a \leq \frac{1}{2}$$

$$\frac{\partial p_2}{\partial a} = \frac{2}{3}t(a-2) < 0$$

$$\frac{\partial D_1}{\partial p_2} = \frac{1}{2t(1-a-b)} > 0$$

$$\frac{\partial D_1}{\partial a} + \frac{\partial D_1}{\partial p_2}\frac{\partial p_2}{\partial a} = \frac{3-5a-b}{6(1-a-b)} + \frac{a-2}{3(1-a-b)} = -\frac{3a+b+1}{6(1-a-b)} < 0$$

Equilibrium: $a^* = b^* = 0$.

Strategic effect stronger than direct effect.
Maximum differentiation in equilibrium.

Social optimum:

No quantity effect. Social planner wants to minimize total transportation costs. (Kaldor-Hicks vs. Pareto)

In social optimum, the two firms split the market and locate in the middle of each segment: ¼ and ¾.

In equilibrium, product variants are too different.

- Crucial assumption: convex transportation costs.

- Also other equilibria, but they are in mixed strategies.
  [Bester *et al.*, ‘‘A Noncooperative Analysis of Hotelling's Location Game”, *Games and Economic Behavior* 1996]

- Multiple dimensions of variations: Hotelling was almost right
  [Irmen and Thisse, ”Competition in multi-characteristics spaces: Hotelling was almost right”, *Journal of Economic Theory* 1998]

- Head-to-head competition in shopping malls: Consumers poorly informed?

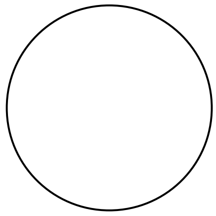
Have we really solved the problem whether or not the equilibrium provision of product variants has too much or too little differentiation?

Too many variants in equilibrium?

A model without location choice.

Focus on firms' entry into the market.

The circular city



Circumference: 1
Consumers uniformly distributed around the circle.
Number of consumers: 1
Linear transportation costs: $t(d) = td$
Unit demand, gross utility = s

Entry cost: f

Unit cost of production: c

Profit of firm i: $\pi_i = (p_i - c)D_i - f$, if it enters,
0, otherwise

Two-stage game:

Stage 1: Firms decide whether or not to enter. Assume entering firms spread evenly around the circle.

Stage 2: Firms set prices.

If *n* firms enter at stage 1, then they locate a distance 1/*n* apart.

Stage 2: Focus on symmetric equilibrium.

If all other firms set price *p*, what then should firm *i* do?

Each firm competes directly only with two other firms: its neighbours on the circle.

At a distance $\tilde{x}$ in each direction is an indifferent consumer:

$$p_i + t\tilde{x} = p + t\left(\frac{1}{n} - \tilde{x}\right)$$

$$\tilde{x} = \frac{1}{2t}\left(p + \frac{t}{n} - p_i\right)$$

Demand facing firm *i*:

$$D_i(p_i, p) = 2\tilde{x} = \frac{1}{n} + \frac{p - p_i}{t}$$

Firm *i*'s problem:

$$\max_{p_i} \pi_i = (p_i - c)\left(\frac{1}{n} + \frac{p - p_i}{t}\right) - f$$

$$\frac{\partial \pi_i}{\partial p_i} = \left(\frac{1}{n} + \frac{p - p_i}{t}\right) - (p_i - c)\frac{1}{t} = 0$$

$$2p_i - p = c + \frac{t}{n}$$

In a symmetric equilibrium, all prices are equal. $\Rightarrow p_i = p$.

$$p = c + \frac{t}{n}$$

Stage 1:
How many firms will enter?

$$D_i = \frac{1}{n}$$

$$\pi_i = (p - c)\frac{1}{n} - f = \frac{t}{n^2} - f$$

$$\pi = 0 \Rightarrow n = \sqrt{\frac{t}{f}}$$

$$\Rightarrow p = c + \frac{t}{\sqrt{t/f}} = c + \sqrt{tf}$$

Condition: Indifferent consumer wants to buy:

$$s \geq p + \frac{t}{2n} = c + \frac{3}{2}\sqrt{tf} \quad \Leftrightarrow \quad f \leq \frac{4}{9t}(s-c)^2$$

Exercise 7.3: What if transportation costs are quadratic?

[Exercise 7.4: What if fixed costs are large?]

<u>Social optimum</u>: Balancing transportation and entry costs.

Average transportation cost: $t\,\frac{1}{2}\tilde{x} = \frac{t}{2}\frac{1}{2n} = \frac{t}{4n}$

The social planner's problem:

$$\min_n \left(nf + \frac{t}{4n} \right)$$

FOC: $f - \frac{t}{4n^2} = 0 \quad \Rightarrow n^* = \frac{1}{2}\sqrt{\frac{t}{f}} < n^e$

Too many firms in equilibrium.

Private motivation for entry: business stealing
Social motivation for entry: saving transportation costs

[Exercise: What happens with n^e/n^* as N (number of consumers) grows?]

Advertising

- informative
- persuasive

Persuasive: shifting consumers' perferences?

Focus on informative advertising.

Hotelling model, two firms fixed at 0 and 1, consumers uniformly distributed across [0,1], linear transportation costs *td*, gross utility *s*.

A consumer is able to buy from a firm if and only if he has received advertising from it.

φ_i – fraction of consumers receiving advertising from firm *i*

Advertising costs: $A_i = A_i(\varphi_i) = \frac{a}{2}\varphi_i^2$

Potential market for firm 1: φ_1.
Out of these consumers, a fraction $(1 - \varphi_2)$ have not received any advertising from firm 2.
The rest, a fraction φ_2 out of φ_1, know about both firms.

Firm 1's demand:

$$D_1 = \varphi_1[(1 - \varphi_2) + \varphi_2\left(\frac{1}{2} + \frac{p_2 - p_1}{2t}\right)]$$

A simultaneous-move game.

Each firm chooses advertising and price.

Firm 1's problem:

$$\max_{p_1,\varphi_1} \pi_1 = (p_1 - c)\varphi_1\left[(1-\varphi_2) + \varphi_2\left(\frac{1}{2} + \frac{p_2 - p_1}{2t}\right)\right] - \frac{a}{2}\varphi_1^2$$

Two FOCs for each firm.

FOC[p_1]: $\varphi_1\left[(1-\varphi_2) + \varphi_2\left(\frac{1}{2} + \frac{p_2 - p_1}{2t}\right)\right] - (p_1 - c)\frac{\varphi_1\varphi_2}{2t} = 0$

FOC[φ_1]: $(p_1 - c)\left[(1-\varphi_2) + \varphi_2\left(\frac{1}{2} + \frac{p_2 - p_1}{2t}\right)\right] - a\varphi_1 = 0$

$$\Rightarrow \quad p_1 = \frac{1}{2}(p_2 + c - t) + \frac{t}{\varphi_2}$$

$$\varphi_1 = \frac{1}{a}(p_1 - c)\left[(1-\varphi_2) + \varphi_2\left(\frac{1}{2} + \frac{p_2 - p_1}{2t}\right)\right]$$

Firms are identical $\Rightarrow$ Symmetric equilibrium

$$p = \frac{1}{2}(p + c - t) + \frac{t}{\varphi}$$

$$\Rightarrow\ p = c + t\left(\frac{2}{\varphi} - 1\right)$$

$$\varphi = \frac{1}{a}(p - c)\left[(1 - \varphi) + \varphi\frac{1}{2}\right]$$

$$\varphi = \frac{1}{a}t\left(\frac{2}{\varphi} - 1\right)\left(1 - \frac{\varphi}{2}\right)$$

$$\Rightarrow\ \varphi = \frac{2}{1 + \sqrt{\frac{2a}{t}}}$$

Condition: $\frac{a}{t} \geq \frac{1}{2}$

$$\Rightarrow p = c + \sqrt{2at}$$

Condition: $s \geq c + t + \sqrt{2at}$ $(\geq c + 2t)$

- $\frac{\partial\varphi}{\partial a} < 0, \quad \frac{\partial p}{\partial a} > 0$

Firms' profit:

$$\pi = \frac{2a}{\left(1+\sqrt{\frac{2a}{t}}\right)^2}$$

- $\frac{\partial \pi}{\partial t} > 0; \quad \frac{\partial \pi}{\partial a} > 0!$

An increase in advertising costs increases firms' profits.

Two effects of an increase in *a* on profits:

A direct, negative effect.
An indirect, positive effect: $a\uparrow \;\rightarrow\; \varphi\downarrow \;\rightarrow\; p\uparrow$

Firms profit collectively from more expensive advertising.

Crucial assumption: convex advertising costs.

What about the market for advertising?
[Kind, Nilssen, & Sørgard, *Journal of Media Economics* 2007]

Social optimum

Average transportation costs

among fully informed consumers: $t/4$.

among partially informed consumers: $t/2$.

The social planner's problem:

$$\max_{\varphi} \varphi^2\left(s-c-\frac{t}{4}\right)+2\varphi(1-\varphi)\left(s-c-\frac{t}{2}\right)-2\frac{a}{2}\varphi^2$$

$$\varphi^* = \frac{2(s-c)-t}{2(s-c)+2a-\frac{3}{2}t}$$

[Condition: $t \leq 2(s-c)$]

Special cases:

(i) $\frac{a}{t} \to \frac{1}{2}$:

$$\varphi^e \to 1$$

$$\varphi^* \to 1-\frac{t}{4(s-c)-t} < 1$$

Too much advertising in equilibrium

(ii) $\frac{a}{t} \to \infty$:

$$\varphi^e \to 0$$

$$\varphi^* \to \frac{1}{1+\frac{a}{s-c}} > 0$$

Too little advertising in equilibrium